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Construction: Join OR, AQ, RC, QB & EF

By taking BC as diameter and $\angle BEC = \angle BFC = 90^{\circ}$, we can construct a circle passes through B, C, E, F. Proof: $\angle EBF = \angle ECF$ (Angle on the same segment EF) ------(1) $\angle APQ = \angle ACQ = \angle ABQ$ (Angle on the same segment AQ) ------(2) As $\langle ECF = \langle ACQ, so \langle EBF = \langle APQ = \langle ECF = \langle ACQ -----(3) \rangle$ In triangle OAB and triangle RAP

<OBA = <RPA (from 3)

<OAB = <RAP (Ver cally Opposite angle)

Hence ΔOAB~ΔRAP -----(5)

In triangle OBQ, <FBQ=<FBO and BF \perp OQ

 Δ OBQ is an isosceles triangle with BO = OQ and OF = FQ.

<BOQ = <BQO (Opposite angle of equal sides). -----(6)

In triangle ORQ, OF = FQ and $RF \perp OQ$

 ΔROQ is an isosceles triangle with OR = RQ.

Hence ER is the angle bisector of <ORQ

So, <ORF = <QRF -----(7)

In triangle ROB and triangle RAP

<OBR = <RPA (from 3)

<ORB = <ARP (from 7)

 $\Rightarrow \Delta \text{ROB} \sim \Delta \text{RAP} ------(8)$

From (5) and (7) $\triangle ROB \sim \triangle OAB$

Hence Ra o of their sides are equal

OB/AB = RB/OB

 \Rightarrow OB²=AB×RB (PROVED)