

Construction: Join OR, AQ , RC, QB \& EF
By taking BC as diameter and $\angle B E C=\angle B F C=90^{\circ}$, we can construct a circle passes through $\mathrm{B}, \mathrm{C}, \mathrm{E}, \mathrm{F}$.
Proof: $\angle \mathrm{EBF}=\angle \mathrm{ECF}$ (Angle on the same segment EF) $\qquad$
$\angle A P Q=\angle A C Q=\angle A B Q$ (Angle on the same segment AQ)
$\mathrm{As} \varangle \mathrm{ECF}=\angle \mathrm{ACQ}$, so $\angle \mathrm{EBF}=\angle \mathrm{APQ}=\angle \mathrm{ECF}=\angle \mathrm{ACQ}$
In triangle OAB and triangle RAP

```
\angleBA = }<\mathrm{ RPA (from 3)
<AB = <RAP (Ver cally Opposite angle)
```

Hence $\triangle \mathrm{OAB} \sim \mathrm{RAP}$ $\qquad$
In triangle $\mathrm{OBQ}, \varangle \mathrm{FBQ}=\varangle \mathrm{BO}$ and $\mathrm{BF} \perp \mathrm{OQ}$
$\triangle O B Q$ is an isosceles triangle with $\mathrm{BO}=\mathrm{OQ}$ and $\mathrm{OF}=\mathrm{FQ}$.
$\triangleleft \mathrm{BOQ}=\triangle \mathrm{BQO}$ (Opposite angle of equal sides).
In triangle $\mathrm{ORQ}, \mathrm{OF}=\mathrm{FQ}$ and $\mathrm{RF} \perp \mathrm{OQ}$
$\triangle R O Q$ is an isosceles triangle with $O R=R Q$.
Hence $E R$ is the angle bisector of $\angle O R Q$
So, $\lessdot$ RF $=\measuredangle Q R F$ $\qquad$
In triangle ROB and triangle RAP

$$
\begin{array}{ll}
<O B R=\angle R P A & \text { (from 3) } \\
<O R B=\angle A R P & (\text { from } 7) \tag{8}
\end{array}
$$

$\Rightarrow \triangle$ ROB $\sim$ RAP
From (5) and (7) $\quad \triangle \mathrm{ROB} \sim \triangle \mathrm{OAB}$
Hence Ra of their sides are equal
OB/AB =RB/OB
$\Rightarrow \mathrm{OB}^{2}=\mathrm{AB} \times \mathrm{RB}$ (PROVED)

